DPT/00/78
IPPP/00/12

Lifetimes of b-flavoured hadrons

Fulvia De Fazio†

† Institute for Particle Physics Phenomenology, University of Durham, DH1 3LE, UK

Abstract. I discuss the heavy quark expansion for the inclusive widths of heavy-light hadrons, which predicts quite well the experimental ratios of B_q meson lifetimes. As for Λ_b, current determinations of $\mathcal{O}(m_b^{-3})$ contribution to $\tau(\Lambda_b)$ do not allow to explain the small measured value of $\tau(\Lambda_b)/\tau(B_d)$. As a final topic, I discuss the implications of the measurement of the B_c lifetime.

1. Lifetimes of heavy-light hadrons

Inclusive particle widths describe the decay of the particle into all possible final states with given quantum numbers f. For weakly decaying heavy-light $Q\bar{q}$ (Qqq) hadrons H_Q, the *spectator model* considers only the heavy quark Q as active in the decay, the light degrees of freedom remaining unaffected. Hence, all the hadrons containing the same heavy quark Q should have the same lifetime; this picture should become accurate in the $m_Q \to \infty$ limit, when the heavy quark decouples from the light degrees of freedom. However, the measurement of beauty hadron lifetime ratios [1]:

$$\frac{\tau(B^-)}{\tau(B_d)} = 1.066 \pm 0.02 \ , \ \frac{\tau(B_s)}{\tau(B_d)} = 0.99 \pm 0.05 \ , \ \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.794 \pm 0.053 \quad (1)$$

shows that $\tau(\Lambda_b)/\tau(B_d)$ significantly differs from the spectator model prediction.

A more refined approach consists in computing inclusive decay widths of H_Q hadrons as an expansion in powers of m_Q^{-1} [2]. Invoking the optical theorem, one can write $\Gamma(H_Q \to X_f) = 2Im\langle H_Q|\hat{T}|H_Q\rangle/2M_{H_Q}$, with $\hat{T} = i\int d^4x T[\mathcal{L}_w(x)\mathcal{L}_w^\dagger(0)]$ the transition operator describing the heavy quark Q with the same momentum in the initial and final state, and $\mathcal{L}_w$ the effective lagrangian governing the decay $Q \to X_f$. An operator product expansion of $\hat{T}$ in the inverse mass of the heavy quark allows to write: $\hat{T} = \sum_i C_i \mathcal{O}_i$, with the local operators $\mathcal{O}_i$ ordered by increasing dimension, and the coefficients C_i proportional to increasing powers of m_Q^{-1}. As a result, for a beauty hadron H_b the general expression of the width $\Gamma(H_b \to X_f)$ is:

$$\Gamma(H_b \to X_f) = \Gamma_0\Big[c_3^f\langle\bar{b}b\rangle_{H_b} + \frac{c_5^f}{m_b^2}\langle\bar{b}ig_s\sigma\cdot Gb\rangle_{H_b} + \sum_i \frac{c_6^{f(i)}}{m_b^3}\langle\mathcal{O}_i^6\rangle_{H_b} + \mathcal{O}\Big(\frac{1}{m_b^4}\Big)\Big] \ , \quad (2)$$

with $\langle O\rangle_{H_b} = \dfrac{\langle H_b|O|H_b\rangle}{2M_{H_b}}$, $\Gamma_0 = \dfrac{G_F^2 m_b^5}{192\pi^3}|V_{qb}|^2$ and V_{qb} the relevant CKM matrix element.

The first operator in (2) is $\bar{b}b$, with dimension $D=3$; the chromomagnetic operator $\mathcal{O}_G = \bar{b}\frac{g}{2}\sigma_{\mu\nu}G^{\mu\nu}b$, responsible of the heavy quark-spin symmetry breaking, has $D=5$; the operators O_i^6 have $D=6$. In the limit $m_b \to \infty$, the heavy quark equation of motion allows to write:

$$\langle \bar{b}b\rangle_{H_b} = 1 + \frac{\langle \mathcal{O}_G\rangle_{H_b}}{2m_b^2} - \frac{\langle \mathcal{O}_\pi\rangle_{H_b}}{2m_b^2} + \mathcal{O}\Big(\frac{1}{m_b^3}\Big), \tag{3}$$

with $\mathcal{O}_\pi = \bar{b}(i\vec{D})^2 b$ the heavy quark kinetic energy operator. When combined with (3), the first term in (2) reproduces the spectator model result. $\mathcal{O}(m_b^{-1})$ terms are absent [3, 4] since $D=4$ operators are reducible to $\bar{b}b$ by the equation of motion. Finally, the operators $\mathcal{O}_G$ and $\mathcal{O}_\pi$ are spectator blind, not sensitive to light flavour. Their matrix elements can be determined from experimental data; as a matter of fact, defining $\mu_G^2(H_b) = \langle \mathcal{O}_G\rangle_{H_b}$ and $\mu_\pi^2(H_b) = \langle \mathcal{O}_\pi\rangle_{H_b}$, one has: $\mu_G^2(B) = 3(M_{B^*}^2 - M_B^2)/4$, while $\mu_G^2(\Lambda_b) = 0$ since the light degrees of freedom in the Λ_b have zero total angular momentum relative to the heavy quark. Moreover, from the mass formula: $M_{H_b} = m_b + \bar{\Lambda} + \dfrac{\mu_\pi^2 - \mu_G^2}{2m_b} + \mathcal{O}(m_b^{-2})$, with $\bar{\Lambda}$, μ_π^2 and μ_G^2 independent of m_b, and from the experimental data, one can infer $\mu_\pi^2(B_d) \simeq \mu_\pi^2(\Lambda_b)$, as confirmed by QCD sum rule estimates [6].

The $\mathcal{O}(m_b^{-3})$ terms in (2) come from four-quark operators, accounting for the presence of the spectator quark in the decay. Their general expression is [5]:

$$\begin{aligned} O^q_{V-A} &= (\bar{b}_L\gamma_\mu q_L)(\bar{q}_L\gamma_\mu b_L) \qquad & T^q_{V-A} &= (\bar{b}_L\gamma_\mu t^a q_L)(\bar{q}_L\gamma_\mu t^a b_L) \\ O^q_{S-P} &= (\bar{b}_R q_L)(\bar{q}_L b_R) \qquad & T^q_{S-P} &= (\bar{b}_R t^a q_L)(\bar{q}_L t^a b_R)\ . \end{aligned} \tag{4}$$

Their matrix elements over B_q can be parametrized as:

$$\langle O^q_{V-A}\rangle_{B_q} = \langle O^q_{S-P}\rangle_{B_q}\Big(\frac{m_b+m_q}{M_{B_q}}\Big) = f_{B_q}^2\frac{M_{B_q}}{8}, \quad \langle T^q_{V-A}\rangle_{B_q} = \langle T^q_{S-P}\rangle_{B_q} = 0\ \ , \tag{5}$$

f_{B_q} being the B_q decay constant. As for Λ_b, one can write:

$$\langle \tilde{O}^q_{V-A}\rangle_{\Lambda_b} = f_B^2 M_B\ r/48, \quad \langle O^q_{V-A}\rangle_{\Lambda_b} = -\tilde{B}\langle \tilde{O}^q_{V-A}\rangle_{\Lambda_b} \tag{6}$$

with $\tilde{O}^q_{V-A} = (\bar{b}_L\gamma_\mu b_L)(\bar{q}_L\gamma_\mu q_L)$. In the valence quark approximation $\tilde{B}=1$.

Actually, with the computed values of the Wilson coefficients in (2), only large values of the parameter r in (6) (namely $r \simeq 3-4$) could explain the observed difference between $\tau(\Lambda_b)$ and $\tau(B_d)$. This, however, seems not to be the case.

2. $\langle \tilde{O}^q_{V-A}\rangle_{\Lambda_b}$ from QCD sum rules

The parameter r in (6) can be determined using quark models or lattice QCD [7]. HQET QCD sum rules allow to estimate it from the correlator:

$$\Pi_{CD} = (1+\not{v})_{CD}\Pi(\omega,\omega') = i^2\int dx dy\ e^{i\omega v\cdot x - i\omega' v\cdot y}\langle 0|T[J_C(x)\tilde{O}^q_{V-A}(0)J_D(y)]|0\rangle \tag{7}$$

between Λ_b interpolating fields $J_{C,D}$ (C, D Dirac indices) [8] and the operator $\tilde{O}^q_{V-A}$; ω (ω') is related to the residual momentum of the incoming (outgoing) current $p^\mu =$

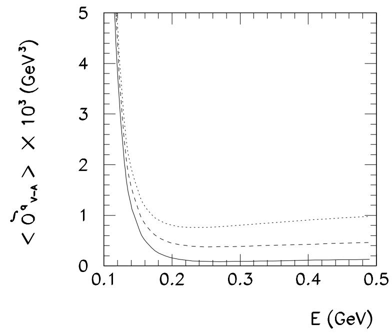

Figure 1. Sum rule for $\langle \tilde{O}^q_{V-A} \rangle_{\Lambda_b}$ as a function of the Borel variable E.

$m_b v^\mu + k^\mu$ with $k^\mu = \omega v^\mu$. The projection of the interpolating fields on the Λ_b state is parametrized by $\langle 0|J_C|\Lambda_b(v)\rangle = f_{\Lambda_b}(\psi_v)_C$ (with ψ_v the spinor for a Λ_b of velocity v).

Saturating the correlator $\Pi(\omega, \omega')$ with baryonic states and considering the low-lying double-pole contribution in the variables ω and ω', one has:

$$\Pi^{had}(\omega, \omega') = \langle \tilde{\mathcal{O}}^q_{V-A} \rangle_{\Lambda_b} \frac{f^2_{\Lambda_b}}{2} \times \frac{1}{(\Delta_{\Lambda_b} - \omega)(\Delta_{\Lambda_b} - \omega')} + \dots \tag{8}$$

with Δ_{Λ_b} defined by $M_{\Lambda_b} = m_b + \Delta_{\Lambda_b}$. Besides, for negative values of ω, ω', Π can be computed in QCD in terms of a perturbative contribution and of vacuum condensates:

$$\Pi^{QCD}(\omega, \omega') = \int d\sigma d\sigma' \frac{\rho_\Pi(\sigma, \sigma')}{(\sigma - \omega)(\sigma' - \omega')} \tag{9}$$

with possible subtractions omitted [9]. The sum rule consists in equating Π^{had} and Π^{QCD}. Moreover, invoking global duality, the contribution of higher resonances and of continuum to Π^{had} can be modeled as the QCD term in the region $\omega \geq \omega_c$, $\omega' \geq \omega_c$, with ω_c an effective threshold. Finally, a double Borel transform to Π^{QCD} and Π^{had} in ω, ω', with Borel parameter E_1, E_2, removes the subtraction terms in (9), improves factorially the convergence of the OPE and enhances the contribution of the low-lying resonances in Π^{had}. Choosing $E_1 = E_2 = 2E$, one gets a sum rule the result of which is depicted in figure 1. Considering the variation with E and the threshold ω_c, one has an estimate of $\langle \tilde{\mathcal{O}}^q_{V-A} \rangle_{\Lambda_b}$:

$$\langle \tilde{\mathcal{O}}^q_{V-A} \rangle_{\Lambda_b} \simeq (0.4 - 1.20) \times 10^{-3}\ GeV^3 \quad , \tag{10}$$

corresponding to $r \simeq 0.1 - 0.3$ [9]. The same calculation gives $\tilde{B} \simeq 1$. This result produces $\tau(\Lambda_b)/\tau(B_d) \geq 0.94$, at odds with the experimental result. The discrepancy discloses exciting perspectives both from experimental and theoretical sides [10].

3. B_c lifetime

A different hadronic system, whose lifetime can be determined by OPE-based methods, is the B_c meson, observed at Fermilab with mass $M_{B_c} = 6.40 \pm 0.39 \pm 0.13\ GeV$ and lifetime $\tau_{B_c} = 0.46\ \pm^{0.18}_{0.16} \pm 0.03\ ps$ [11]. Like quarkonium states, B_c can be treated in a non relativistic way, but unlike them it can decay only weakly, with the main decay

mechanisms induced by the quark transitions $b \to cW^-$, $\bar{c} \to \bar{s}W^-$ and $\bar{c}b \to W^-$ (annihilation). Predictions for τ_{B_c} spread in the range $0.4 - 1.2$ ps [12, 13, 14]. In the $m_b, m_c \to \infty$ limit one would have $\Gamma_{B_c} = \Gamma_{b,spec} + \Gamma_{c,spec}$. Corrections to this result can be computed using an OPE organized in powers of the heavy quark velocity [14]. The result is: $\tau_{B_c} \simeq 0.4 - 0.7$ ps, together with the prediction of the dominance of charm transitions; as a matter of fact, b-decay dominance would imply a larger lifetime: $\tau_{B_c} = 1.1 - 1.2$ ps [13]. Hence, the measurement of τ_{B_c} provides us with the first hints on the underlying dynamics in this meson. For this system, it is interesting to investigate the validity of the non relativistic approximation: actually, one estimates $\langle k^2 \rangle / m_c^2 \simeq 0.43$, where $\langle k^2 \rangle$ is the average squared momentum of the charm quark, implying possible deviations from the non relativistic limit [15].

4. Conclusions

$1/m_Q$ expansion can be used to compute inclusive widths of heavy-light hadrons. A QCD sum rule calculation of the matrix element $\langle \tilde{O}^q_{V-A} \rangle_{\Lambda_b}$ contributing to $\mathcal{O}(m_b^{-3})$ to the Λ_b lifetime gives the result: $\tau(\Lambda_b)/\tau(B_d) \geq 0.94$, thus implying that such a correction does not explain the observed difference between $\tau(\Lambda_b)$ and $\tau(B_d)$. Finally, the measurement of B_c lifetime already enlightens some aspects of the quark dynamics in this meson.

Acknowledgments

I thank P. Colangelo for collaboration on the topics discussed above. I acknowledge for support the EU-TMR Programme, Contract No. CT98-0169, EuroDAΦNE.

References

[1] LEP B lifetime group, http://wwwcn.cern.ch∼claires/lepblife.html.
[2] I. Bigi *et al.*, in S. Stone (ed.), B Decays, 2nd ed., World Scientific, pag.132.
[3] J. Chay *et al* ., *Phys. Lett.* **B 247** (1990) 399.
[4] I. Bigi *et al* ., *Phys. Lett.* **B 293** (1992) 430; **297** (1993) 477 (E).
[5] M. Neubert and C.T. Sachrajda, *Nucl. Phys.* **B 483** (1997) 339.
[6] P. Ball *et al* ., *Phys. Rev.* **D 49** (1994) 2472; P. Colangelo *et al* ., *Phys. Rev.* **D 54** (1996) 4622.
[7] B. Guberina *et al* ., *Z. Phys.* **C 33** (1986) 297; J.L. Rosner, *Phys. Lett.* **B 379** (1996) 267; M. Di Pierro *et al.*, UKQCD Collab. hep-lat/9906031.
[8] E.V. Shuryak, *Nucl. Phys.* **B 198** (1982) 83.
[9] P. Colangelo and F. De Fazio, *Phys. Lett.* **B 387** (1996) 371.
[10] M. B. Voloshin, hep-ph/0011099.
[11] CDF Collab., *Phys. Rev. Lett.* **81** (1998) 2432; *Phys. Rev.* **D58** (1998) 112004.
[12] M. Lusignoli and M. Masetti, *Z. Phys.* **C 51** (1991) 549; P. Colangelo *et al* ., *Z. Phys.* **C 57** (1993) 43; A.Y. Anisimov *et al* ., *Phys. Lett.* **B452** (1999) 129.
[13] C. Quigg, in *Proceedings of the Workshop on B Physics at Hadron Accelerators*, Snowmass, Colorado 1993, ed. P. McBride and C.S. Mishra, pag. 439.
[14] M. Beneke and G. Buchalla, *Phys. Rev.* **D53** (1996)4991.
[15] P. Colangelo and F. De Fazio, *Mod. Phys. Lett.* **A14** (1999) 2303; *Phys. Rev.* **D61** (2000) 034012.